AN ANALYTICAL MODEL FOR THE PREDICTION OF THE RESISTANCE OF T-STUBS WITH FOUR BOLTS IN A ROW

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INTRODUCTION

In case of bolted connections, the main components that contribute to the deformability of the joint are: the column flange in bending, the end-plate in bending and the angles in tension which are modelled as an equivalent T-stub in tension. In technical literature, several experimental and theoretical works deal with the characterization of the behavior of T-stubs fastened by means of two bolts \cite{4,7,8} but, the case of T-stubs with more than two bolts per row is not considered, even though it is a sub-assemblage that is applied in many practical situations \cite{6}. In this paper, in order to provide a contribution to improve the model given in EC3, extending it to cases not currently covered in the code, attention is focused on the experimental and theoretical analysis of T-stubs with four bolts per row, following three steps: experimental investigation, finite element modelling and proposal of a mechanical model for the prediction of the T-stub plastic resistance.

1 PREVIOUS STUDIES

Dealing with the behaviour of the T-stub with four bolts in one row some studies have already been carried out in \cite{3}, where the basic formulas to define the resistance of the T-stubs have been determined. In particular, within the approach proposed by \cite{3}, in analogy with the classical T-stub theory, a simplified beam model that extends the formulations given by EC3 to the case of T-stub with four bolts in one row has been developed. In particular, based on the assumptions of rigid-plastic behavior of the steel and of elastic distribution of the forces in the bolts up to failure, the equations reported in Table 1 have been obtained.

![Table 1 – Resistance of T-stubs with four bolts in one row](image)

<table>
<thead>
<tr>
<th>Failure Mode</th>
<th>T-stub with two bolts</th>
<th>T-stub with 4 bolts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode-1</td>
<td>$F_1 = \frac{(8n - 2e_w)M_{pl,1,Rd}}{2mn - e_w(m + n)}$</td>
<td>$F_1 = \frac{(8n - 2e_w)M_{pl,1,Rd}}{2mn - e_w(m + n)}$</td>
</tr>
<tr>
<td>Mode-2</td>
<td>$F_2 = \frac{2M_{pl,2,Rd} + n \sum B_{l,Rd}}{m + n}$</td>
<td>$F_2 = 2M_{pl,2,Rd} + \sum \frac{B_{l,Rd}}{2} \left( \frac{n_1^2 + 2n_2^2 + n_1n_2}{n_1 + n_2} \right)$</td>
</tr>
<tr>
<td>Mode-3</td>
<td>$F_3 = \sum B_{l,Rd}$</td>
<td>$F_3 = \sum B_{l,Rd}$</td>
</tr>
</tbody>
</table>

The symbols have the same meanings given in EC3, excepting $n_1$ which is the distance between the two bolts and $n_2$ which is the distance of the external bolt from the free edge. It is possible to note that the application of the equations obtained by \cite{3} needs the definition of appropriate effective lengths that will be topic of discussion in the next sections.

2 EXPERIMENTAL ACTIVITY

2.1 Description of the layout

As first step of the research activity, an experimental program has been carried out at the laboratory on materials and structures of the University of Coimbra. In particular, three T-stubs with four bolts designed to fail according to different collapse mechanisms have been tested. All the specimens have been realized starting from steel plates made of S355 steel fastened by means of M12 bolts...
made of 8.8 class [2]. In order to favour the development of the three typical failure modes, the specimens have been designed according to the following criteria:

- Test 1: the T-stub has been designed in order to induce the development of a weak plate-strong bolt mechanism realized with a plate of 10 mm;
- Test 2: the T-stub has been designed in order to develop a mechanism type-2 realized with a plate of 20 mm;
- Test 3: the T-stub has been designed to fail according to a mechanism type-3; bolts are located very close to the T-stub web in order to induce the bolt failure mechanism (Fig. 1).

All the tests have been carried out by means of the rig machine developed at the University of Coimbra. The machine is composed by the assemblage of a series of steel profiles that are used to counteract the load applied by an hydraulic cylinder (Maximum load 1000 kN, Stroke 280 mm). The loads have been applied under displacement control at the constant velocity of 0.02 mm/s in order to carry out quasi-static tests. During the tests, four LVDTs on both sides of the T-stub are used to measure the displacements. In particular, the average value of the gap opening and the inner bolts elongation have been controlled during the tests.

### 2.2 Experimental results

The first test, according to the adopted design criteria, was characterized by a strong bolt and weak plate behaviour. From Fig. 2a), it can be noted that the plate of Test-1 exhibited a significant plastic engagement, with a deformed shape characterized by the development of two plastic hinges in correspondence of the T-stub web and two plastic hinges in correspondence of the bolt lines.

In this test, the premature fracture of the bolts was observed before the complete development of the plastic hinges on the T-stub flange plate. In particular, the failure has first concerned the inner bolts and, after a partial loss of the T-stubs resistance, also the outer bolts. Specimen 2 had the same geometrical configuration of the first one, but the flange plate has 20 mm of thickness. In this case the T-stub exhibited a type-2 collapse mode, characterized by the development of a plastic hinge in correspondence of the flange-to-web connection followed by the failure of the bolts. Again in this case, the collapse first arose due to the failure of the inner bolts and, after the partial loss of the load, followed by the outer bolts.

Finally, in case of Test-3, in agreement with the design procedure, for a value of the load corresponding approximately to the sum of the resistances of the four M12 bolts in tension, the failure arose due to the fracture of the bolt shaft. In Fig. 3a) the values of the loads recorded by the
load cell versus the average displacements recorded by the two LVDTs located in the gap opening, are reported for the three specimens.

3 FINITE ELEMENT MODEL

In order to extend the results of the experimental campaign, a Finite Element Model has been carried out in Abaqus code. In particular, the FE model has been developed with two scopes. The first one is to evaluate the yield line patterns arising in the plate, which are necessary to define the values of the effective lengths to be employed in simplified calculations. The second one is the extension of the experimental sample by generating a parametric analysis which is crucial to verify the mechanical model. The FE model is constituted by four parts: the rigid support and its web, the tested T-stub and the bolts. Bolts and plates materials’ properties have been described by means of an elastic-plastic isotropic model. The rigid support has been simulated by adopting an infinitely elastic material with very high value of the stiffness. Concerning the element type, a 8-node linear brick with reduced integration and instability mesh control has been adopted (C3D8R). The mesh size has been defined by carrying out a sensitivity analysis in the preliminary phase, by accounting also for the existing guidelines on the topic[5].

The interaction among the various elements has been defined according to a surface-to-surface formulation with finite sliding. In the normal direction a “hard contact” has been used, while in the tangential direction a friction coefficient equal to 0.2 has been defined. In order to reduce the computational time, half specimen has been modelled accounting for the symmetry. In Fig.3 the results of the FE analysis are reported both in terms of response at failure and in terms of force-displacement curve. It is possible to appreciate that the failure mode predicted analytically, is faithfully reproduced by the model and that the response is well reproduced in terms of stiffness, resistance and ductility supply of the T-stub.

As aforesaid, preliminarily to the development of the mechanical model, the finite element model has been used to evaluate the shape of the yield line families, which is a fundamental requirement to define the values of the effective lengths needed in the simplified calculations. To this scope, in order to point out the different shapes of the yield line patterns of four-bolts T-stub, several FE models with different geometry once with two-bolts and once with four-bolts have been generated. As a result of this analysis the following considerations can be made. In case of mechanism type-1, the second bolt row is not effective and, therefore, the failure mechanisms have no substantial differences with respect to classical ones. Conversely, in case of mechanism type-2, while the beam pattern is still unchanged, the kinematic mechanism in case of non-circular pattern is substantially different. In fact, in such case, the external bolt row forces the yield lines to pass through the bolts leading to a shape more similar to the one classically defined in case of Mechanism type-1. Therefore, from the observation of the FE model results it is clear that in case of 4-bolts T-stubs a
recalibration of the classical model is needed. To this scope, in the next section, a new analytical formulation defining the effective length is developed.

![Mec-1 – Non-Circular Pattern](image1)
![Mec-1 – Non-Circular Pattern](image2)
![Mec-2 – Non-Circular Pattern](image3)
![Mec-2 – Non-Circular Pattern](image4)

**Fig. 4 – Yield linear patterns from FEM**

4 **ANALYTICAL MODEL**

The definition of the effective lengths, necessary to obtain the plastic resistance of the T-stub with four bolts, has been obtained according to the classical approach already proposed in [8]. In particular, the method has been applied by following these steps:

1. Individuation of a possible yield line pattern;
2. Definition of the parameters characterizing the shape of the yield line family;
3. Evaluation for each yield line of the rotation due to an assigned value of the external displacement;
4. Evaluation of internal and external work;
5. Minimization of the energy dissipated by the yield lines in order to find the value of the collapse load.

Starting from the results on the previous analysis of the yield lines, it has been realized that, in case of T-stub with four bolts in one row, the only pattern that cannot be modelled by means of the classical theory [8] is the mechanism type-2 with non-circular pattern. Therefore, for this case, the yield line method has been applied by assuming the distribution of the yield lines deriving from the FE simulation depicted in Fig. 5. Under this hypothesis, it is easy to recognize that the assigned pattern is characterized by two parameters: the angles $\alpha$ and $\beta$ (Fig. 5).

![Fig. 5 – Assumed Yield Line Pattern](image5)

Therefore, assuming that on the yield lines act the flexural plastic resistance of the plate per unit of length $m_{pl,Rd}$, the bolt plastic resistance $B_{Rd}$, and the external force and the internal and external works can be written as follows:

$$T(m + n) = \sum_{i=1}^{5} m_{pl,Rd} l_i \theta_i + \sum B_{Rd} n$$  

(1)
where \( l_i \) is the length of the \( i \)-th yield line and \( \theta_i \) is its rotation. It is evident that in order to define the value of the effective length it is necessary to find the minimum collapse load \( T \) by minimizing the right hand side of Eq.(1). In particular, as far as the bolts resistance does not depend on parameters \( \alpha \) and \( \beta \), it means minimizing the work done by the plastic hinges. Therefore, in order to obtain the value of the minimum collapse load, the equation (1) has been solved numerically by developing a user routine in the software Mathematica 5.1. The developed routine provides for an assigned couple of values of \( m \) and \( n \), the values of \( \alpha \) and \( \beta \) that minimize the energy expressed by Eq. (1) and, therefore, the value of the effective length. To this scope, 10.000 combinations of values \( m \) and \( n \) varying in the range from 10 mm to 1000 mm have been generated. From the results, by means of a multiple regression analysis of the data, the following expression of the effective width has been obtained with correlation coefficient \( R^2 \) equal to 0.99:

\[
l_{eff, np} = 5.685m + 5.867n
\]  

(2)

Based on the obtained results, the authors’ proposal is to define the resistance of the T-stub with four bolts in one row by adopting the formulas reported in Table 1 [3] and the values of the effective lengths given by the classical theory [8], made exception for the case of mechanism type-2 and non-circular pattern, where the effective length should be defined by Eq.2.

### 5 PARAMETRIC ANALYSIS

In order to check the accuracy of the proposed model for the prediction of the resistance of T-stubs with four bolts in one row, a parametric analysis has been developed in ABAQUS by employing the FE model previously described. In particular, the analysis has been carried out by generating a sample of cases with different geometrical characteristics. Twenty-four models have been defined, twelve of them with a wide flange (1200 mm width) and the others twelve with a narrow flange (200 mm). The first group of models has been defined aiming to promote the development of non-circular patterns, while the second group has been defined aiming to promote the development of beam patterns. For each group the following combinations of \( m, n_1 \) and \( n_2 \) have been defined:

1. \( m = 40 \text{ mm}; n_1 = 80 \text{ mm}; n_2 = 35 \text{ mm} \)
2. \( m = 60 \text{ mm}; n_1 = 60 \text{ mm}; n_2 = 35 \text{ mm} \)
3. \( m = 40 \text{ mm}; n_1 = 60 \text{ mm}; n_2 = 55 \text{ mm} \)

In addition, for each combination of \( m, n_1 \) and \( n_2 \), in order to develop collapses belonging to all the possible failure mechanisms, four values of the thickness have been considered: 5 mm, 10 mm, 15 mm and 20 mm. The steel composing the plates is S355 and the bolts are M12, 8.8 grade.

In order to compare the values obtained with the analytical model defined in previous section and the results coming from this parametric analysis, it is necessary to define the plastic resistance for the T-stubs modelled in ABAQUS. To this scope, it is considered that the model presented in EC3 provides a value of the plastic resistance corresponding to the knee that can be determined as a value 1.5 times greater than the resistance corresponding to the yielding [1].

Therefore, in Table 2 the geometrical parameters, the collapse mechanisms, the resulting yield line pattern and the numerical and analytical values are compared. It can be observed that the prediction provided by the model appears sufficiently accurate with an average value of the Model/FEM ratio equal to 1.078 and a standard deviation equal to 0.19. It is useful to observe that in the cases in which the mechanism is type-2 and the yield line pattern is non-linear, the adoption of Eq.2. provided by EC3 would lead to a substantial underestimation of the T-stub plastic resistance. On the sample investigated, the developed model seems to give sufficiently accurate results.

**Table 2 – Model vs FEM**

<table>
<thead>
<tr>
<th>( R ) [( \text{mm} )]</th>
<th>( s ) [( \text{mm} )]</th>
<th>( m ) [( \text{mm} )]</th>
<th>( n_1 ) [( \text{mm} )]</th>
<th>( n_2 ) [( \text{mm} )]</th>
<th>( l_{cp} ) [( \text{mm} )]</th>
<th>( \text{YLP} )</th>
<th>( F_{c,t, MOD} ) [( \text{kN} )]</th>
<th>( F_{c,t, FEM} ) [( \text{kN} )]</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1200</td>
<td>5</td>
<td>40</td>
<td>80</td>
<td>35</td>
<td>251.2</td>
<td>1</td>
<td>CP</td>
<td>27.87</td>
<td>33.37</td>
</tr>
<tr>
<td>1200</td>
<td>5</td>
<td>60</td>
<td>60</td>
<td>35</td>
<td>358.75</td>
<td>1</td>
<td>NCP</td>
<td>26.53</td>
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<tr>
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<td>5</td>
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<td>60</td>
<td>55</td>
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<td>CP</td>
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<tr>
<td>1200</td>
<td>10</td>
<td>40</td>
<td>80</td>
<td>35</td>
<td>906.5</td>
<td>2</td>
<td>NCP</td>
<td>91.43</td>
<td>80.27</td>
</tr>
</tbody>
</table>
6 CONCLUSIONS

In this work, the results of a theoretical and experimental analysis dealing with the prediction of the plastic resistance of T-stubs with four bolts in one row have been presented. Within the work, by exploiting a FE model calibrated on the results of three experimental tests, it has been evidenced that the failure mechanism most affected by the second bolt row is the type 2. Furthermore, by means of the FE model, the yield line patterns for all the failure mechanisms have been individuated, showing that the shape of non-circular yield pattern significantly varies with respect to the classical theory. Finally, a complete analytical model, based on the definition of new values of the effective lengths, has been proposed demonstrating its accuracy by comparing the analytical predictions versus the results obtained from a parametric analysis.

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