DEVELOPMENT OF A SIMPLIFIED PROBABILISTIC METHODOLOGY FOR SAFETY ASSESSMENT OF STABILITY OF STEEL STRUCTURES

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INTRODUCTION

At present, the safety level implied by the specified design rules and partial safety factors γ_M is not homogeneous and consistent throughout Eurocode 3[2], mainly due to lack of guidance and existing databanks containing information on distribution of relevant basic variables and steel properties. Therefore, in this paper, simplified alternatives for the safety assessment are analysed, focusing on stability design rules. The alternatives are briefly summarized; subsequently, a numerical assessment method of the simplified procedures is proposed and finally, a numerical study is performed.

1 EXSISTING ALTERNATIVES FOR SAFETY ASSESSMENT

1.1 Methodological assumption for design resistance

In Section 6 of EN 1990, three different alternatives for the evaluation of the design resistance are proposed, as summarized in *Table 1*:

Method 1	Method 2	Method 3
$R_{d} = \frac{1}{\gamma_{Rd}} R \left\{ \eta \frac{X_{k}}{\gamma_{m}}; a_{d} \right\} i \ge 1$	$R_d = \frac{R_k}{\gamma_M}$	$R_{d} = \frac{1}{\gamma_{M,l}} R\left\{\eta_{i} X_{k,l}; \eta_{i} X_{k,i} \frac{\gamma_{m,l}}{\gamma_{m,i}}; a_{d}\right\} i \ge 1$

Table 1	Alternatives	for	evaluation	of	design	resistance
Tuble 1.	Alternatives	101	evaluation	01	uesign	resistance

Further simplifications may be given for different structural materials but they should not reduce the level of reliability.

Alternatively to Method 1, which is suitable for members composed of multiple materials, the design resistance may be obtained directly from the characteristic value of product or material resistance, without explicit determination of the design values of the individual basic variables (Method 2). The method is applicable to products or members made of a single material and it is also used in connection with Annex D of EN 1990 [1]. It is noted that this simplified approach was used for the evaluation of the design resistance of most failure modes in EN 1993-1-1[2].

Moreover, for structures or structural members that are analysed by non-linear methods and comprise more than one material acting together, Method 3 can be used. Moreover, in section 2.3.4 of EN 1993-1-1[2], it is stated that the evaluation of design resistance should be based on Method 2 or 3.

1.2 Annex D

Annex D of EN 1990 [1] gives a procedure for the safety analysis of "resistance functions", i.e. of "code-type" formulae or methods for the design of structural elements, based on First-Order Reliability Methods. This procedure allows the determination of appropriate values (in a semi-probabilistic design concept) of partial safety factors γ_M on the basis of physical or numerical test results. The flow chart on *Fig. 1* summarizes the method. The procedure identifies two types of uncertainties: i) uncertainty related to the design model, introduced by the coefficient of variation

 V_{δ} ; ii) uncertainty related to the natural randomness, which is presented by the coefficient of variation Vrt.

In step 6 of the procedure, the coefficient of variation of the basic input variables shall be computed. The code proposes two ways to determine the coefficient -i) in case of a simple function (1); and ii) in case of a more complex function (2);

$$V_{rt}^{2} = \sum_{j=1}^{k} V_{x}^{2} \qquad (1) \qquad V_{rt}^{2} = \frac{1}{g(\underline{X}_{m})^{2}} \sum_{j=1}^{k} \left(\frac{\partial g(X_{j})}{\partial X_{j}} \sigma_{j}\right)^{2} \qquad (2)$$
Step 1: Resistance function
 $r_{t} = g_{rt}(X_{1}, X_{2}...X_{n})$
Step 2: Compare experimental
and theoretical values
Step 3: Estimate mean vale of
correction factor b
 $n = \sqrt{n}$

$$\int_{n}^{n} e^{-\sqrt{n}} e^{-\sqrt{n}}$$
Step 5: Analyze compatibility

Fig. 1 Steps of the EN 1990 Annex D procedure for the calculation of design resistances r_d

if the resistance function is

acceptable

1.3 Simplified procedures

 $b = \sum_{i=1}^{n} r_{e,i} r_{t,i} / \sum_{i=1}^{n} r_{t,i}^{2}$

S

a

The above procedure is fairly straightforward, however requires the knowledge of the scatter of every single input parameter, regardless of the actual relevance of this parameter. For certain, individual application fields of steel structure design, e.g. stability or weld design, it may be sensible to use simplified procedures for the calculation of Vrt, which allows one to directly ignore a number of (irrelevant) input parameters altogether. Some alternatives are available in the literature [3, 4] for using Eq. (1) instead of Eq. (2) in Step 6, even when the design function is complex. They are summarized as follows:

Procedure 1(P1) – the procedure has adopted the assumption that the yield stress is the only variable and it is based on Method 2 from Table 1. Furthermore, the coefficient of variation $V_{rt,i}$ of the basic input variables is determined assuming simple function as follows:

$$V_{rt}^{2} = V_{fy}^{2} = \frac{\sigma_{fy}^{2}}{f_{ym}^{2}}$$
(3)

 $r_{d} = \begin{vmatrix} bg_{n}(X_{m}) \exp\left[-k_{d,*}\frac{Q_{n}}{Q} - k_{d,n}\frac{Q_{d}}{Q} - 0.5Q^{2}\right] \\ bg_{n}(X_{m}) \exp\left[-k_{d,*}Q - 0.5Q^{2}\right] \end{vmatrix}$

n ≤ 30

Procedure 2(P2) – the procedure assumes that there could be increased number of variables, however, instead of performing the partial derivatives in case of a complex function, the simplification is made as in Procedure 1;

$$V_{rt}^{2} = \sum_{j=1}^{k} V_{x}^{2} = V_{m}^{2} + V_{cs}^{2} + \dots$$
(4)

2 NUMERICAL VALIDATION OF SIMPLIFIED PROCEDURES P1 AND P2

Dealing with random variables is not as simple as when using deterministic ones. Therefore, a numerical assessment of the simplified procedures was performed. It was based on equation (5). The partial safety factor is given by the ratio between characteristic and design resistance. Considering the procedure of Figure 1, it leads to:

$$\gamma_M^* = \frac{r_{t,nom}}{r_d} = \frac{g_{rt}(\underline{X}_{nom})}{bg_{rt}(\underline{X}_m)\exp(-k_{d,\infty}Q - 0.5Q^2)}$$
(5)

Comparing the procedures with Annex D shows that the terms b, $g_{rt}(\underline{X}_{m})$, $g_{rt}(\underline{X}_{nom})$ are the same regardless of the method used. Therefore, any differences between the two procedures are solely related to the following expression:

$$\frac{1}{\exp\left(-k_{d,\infty}Q - 0.5Q^2\right)}\tag{6}$$

The following methodology is adopted to implement the comparative assessment:

- assumed statistical distributions for the basic input variables are adopted, which are plausible representations of the reality;
- the coefficient of variation of the design model V_{δ} is assumed and varied from 0% to 10%;
- in order to assess the compatibility of the test population, it is split into several sub-sets according to slenderness intervals;
- the "resistance function" formulation for flexural bucking of columns, section 6.3.1 of EC3-1-1[2] is considered;
- a large number of physical experiments are available (at least 100);

In this assessment, the basic variables which were are considered are: i) yield strength (f_y) ; ii) cross-section area (*A*); iii) moment of inertia (*I*); iv) modulus of elasticity – *E*; The following cases are analysed and compared:

variables	P1	P2	P2	P2
f_y	х	х	х	х
Α		Х	Х	Х
Ι			Х	Х
Ε				Х

Table 2Analysed alternatives

where P1 relates to equation (3) and P2 relates to equation (4).



The assessment was performed in two ways – firstly, comparison with Annex D results assuming *the same random variables* as in the procedure P1 or P2: this comparison is aimed at establishing the conservative nature of the simplification(*Fig.2* and *Fig.4*); secondly, comparison of the procedure P1 or P2 with "full" Annex D results using *all the relevant basic input variables as random variables*: this comparison is aimed at establishing the level of safety of the simplified procedures (*Fig.3 and Fig.5*).

When the coefficient of variation of the model is assumed with a fixed value, $V_{\delta}=0.05$. The results are plotted in *Fig.2* and *Fig.4*. The Annex D procedure is always presented in terms of each specimen *i* and the corresponding mean value, whereas for *P1* the result is unique for all specimens (illustrating therefore a horizontal lines in *Fig.2* and *Fig.4*). Moreover, the difference with respect to

the "full" Annex D procedure is studied for variations of the coefficient V_{δ} , as illustrated on *Fig.3* and *Fig.5*. It can be observed that *P1* presents some *unsafe* results when compared to the "full" Annex D (negative values on *Fig.3*). On the contrary, when compared to Annex D (*fy*), i.e., considering *fy* as the only input variable, it is always safe (*Fig.2*).



Furthermore, the number of variables is gradually increased following the assumptions in *Table 2*.In this case, all alternatives of *P2* presented always with safe results when compared to the "full" Annex D with all basic variables (*Fig.4* and *Fig.5*).

However, it is noted that as more variables are included, the differences with Annex D become *higher*. This issue is associated with the fact that simplified procedures are only adding the variability of each parameter. On the contrary, the Annex D procedure using partial derivatives takes into account the variability of each parameter as they appear in the "resistance function".

3 NUMERICAL STUDY

In this section, influence of the input variables, as well as additional analysis of the simplified procedures vs. Annex D procedure (Method 2 of Table 1) were carried out based on a numerical example. Here a series of "numerical tests" were performed, with random input parameters, as basis for a comparison of the design evaluations. The example is based on the flexural buckling formula (resistance function) of EN 1993-1-1. Here, a profile IPE 200 is chosen for the column cross-section. The *nominal* steel grade is S355. The specimens are tested in pure compression for nominal values of the normalized slenderness $\overline{\lambda} = 0.3$, 0.6, 1.0, 1.4, 1.8 and 2.2. For each specimen, the following parameters are measured (i.e. randomly generated for this fictitious "test campaign"): i)Yield strength (f_y); ii) Modulus of elasticity(E); iii) Cross-section (CS- b, h, t_f , t_w); iv) Residual stresses (RS); v) Geometrical imperfections (GI).

	Material J	properties	(Cross-secti	n properties		Imperfections [5]	
	$\mathbf{f}_{\mathbf{y}}$	Е	b	h	$t_{\rm w}$	$t_{\rm f}$	RS	GI
mean	455MPa	210GPa	b _{nom}	h _{nom}	1.025t _{w,nom}	0.975t _{fnom}	$0.2\sigma_{res}$	0.0085L
c.o.v	5.4%	5.0%	0.9%	0.9%	4.0%	3.0%	25.0%	23.5%

Table 3Assumed distributions of basic input variables

Table 3 summarizes the adopted assumptions for the distributions of basic variables, from which random combinations of input parameters were generated for the individual "numerical tests". Firstly, the influence of the included basic variables on the resistance function was studied based on the procedure proposed in Annex D using the partial derivatives only for the respective variables.

Here, partial safety factors $\gamma_{M,i}$ are calculated for each column. Finally, the mean value of all the specimens in each slenderness case is obtained and further plotted on *Fig. 6*. Line $\gamma_{M}=1$ is plotted as a reference value. It is clear that in the high slenderness range, the yield stress is not so important, whereas the modulus of elasticity and the inertia are dominating the behaviour. Observation of Fig. 6 leads one to conclude that not only the yield stress, but also the cross section dimensions are relevant for the evaluation of the partial safety factor.



Fig. 6 Influence of input variables on the evaluation of the partial safety factor based on the Annex D procedure In a second step of the analysis, the simplified procedures were once again compared to the Annex D procedure in Fig. 7. In Figures 7 and 8, and Tables 4 and 5, the obtained results can be seen. The same trends that were shown in Section 2 can be noticed here:





Fig. 8 Comparison of partial safety factors

	Annex D (fy+CS+E)	Annex D (fy)	P1 (fy)	Error
λ=0.3	0.9312	0.9135	0.9237	-0.81%
λ=0.6	0.9231	0.9024	0.9376	1.57%
λ=1.0	0.9914	0.8992	0.9939	0.26%
λ=1.4	1.0672	0.9123	1.0430	-2.26%
λ=1.8	1.1131	0.9292	1.0745	-3.46%
λ=2.2	1.1387	0.9423	1.0932	-4.00%

Table 5 Difference P2(fy+A)

	Annex D (fy+CS+E)	Anne xD (fy+CS)	P2 (fy+A)	Error
λ=0.3	0.9312	0.9309	0.9423	1.19%
λ=0.6	0.9231	0.9187	0.9562	3.58%
λ=1.0	0.9914	0.9517	1.0270	3.59%
λ=1.4	1.0672	1.0058	1.0826	1.44%
λ=1.8	1.1131	1.0427	1.1170	0.36%
λ=2.2	1.1387	1.0638	1.1364	-0.20%

Considering the yield stress as only variable for the simplified procedure may lead to unsafe results for some slenderness ranges when it is compared with the Annex D procedure using all variables.

However it is always safe when it is compared to its respective assumption for basic variables to be included in the Annex D procedure.

Nevertheless, regarding Fig. 8, it is noticed for $\overline{\lambda} = 2.2$, the simplified procedure P2 starts to become unsafe, unlike the observations in the numerical assessment, where the procedure was showing only safe-sided results. This example shows clearly that the assessment is highly dependent on the distributions of variables used and therefore the need of databanks with information on the distributions of basic variables.

When comparing the errors of Tables 4 and 5, once more it was seen that the increased number of variables leads to increased difference between the simplified procedure and Annex D.

Finally, it was attempted to study if it is possible to obtain the experimental results from models with nominal characteristics and further apply the assessment procedure; if so, it would be very useful, since the number of simulations can be significantly reduced. However, when observing the differences between the nominal numerical results and the nominal theoretical, the differences along the buckling curve were not the same, which made impossible to conclude that a model with nominal properties may be used for safety assessment for the time being.

4 CONCLUSIONS

In this paper, different possibilities for safety assessment of design rules focusing on the buckling resistance of steel members were summarized. The study aimed at clarifying the alternatives for safety assessment in order to propose thorough safety assessment procedure to be used in line with Eurocode [2]. This research led to the following main conclusions:

- Simplified procedures which include the variability of the yield strength as the only basic variable (*P1*) may be unsafe for certain slenderness ranges when compared to the Annex D procedure considering all relevant basic input variables.
- When geometrical properties were included, *P2* showed results mostly on the safe side. However, a clear trend of increasing the error between *P2* and the "full" Annex D with increased number of variables considered was noted in both the numerical validation and the example presented. The right balance between "simplicity" and omission of parameters must therefore be found if one of the simplified procedures shall be adopted.

Although all the numerical comparisons were performed only on the basis of the flexural buckling of columns, it will be verified if the conclusions are equally valid to other stability phenomena (LTB of beams, TB and LTB of columns and the buckling resistance of beam-columns), which is the next step of this research.

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