ANALYSIS OF CYLINDRICALLY CURVED STEEL PANELS
An imperfection sensitivity study

João Pedro Martins[a], Darko Beg[b], Franc Sinur[b], Luís Simões da Silva[a]

[a] ISISE, University of Coimbra, Department of Civil Engineering, Portugal
jpmartins@dec.uc.pt, luisss@dec.uc.pt
[b] University of Ljubljana, Faculty of Civil and Geodetic Engineering, Slovenia
darko.beg@fgg.uni-lj.si, franc.sinur@fgg.uni-lj.si

INTRODUCTION

Shell-like structures are very well known to be very prone to stability problems and, also very sensitive to initial imperfections which lower non-negligibly their ultimate strength. On one hand, cylindrically curved panels may be seen as a subgroup of shell structures and therefore share the disadvantages of such structural solution. On the other hand, cylindrically curved panels may be seen as curved plates. The reality lies somewhere between these two limits.
Since curved panels are neither flat nor full revolution shells, rules and guidelines for estimating equivalent geometric imperfections are inexistent. This problem becomes more serious when preliminary results have shown a high sensitivity to initial geometric imperfections.
In the framework of Eurocodes, rules defining how to proceed to model geometric imperfections when using numerical tools (*e.g.* when using the finite element method) are given by EN1993-1-5 and EN1993-1-6. However, cylindrically curved panels fall outside the scope of these standards.
This paper focus on fully nonlinear numerical analysis of cylindrically curved panels with different patterns for the geometric imperfections, all based on several eigenmode shapes.

1 ON THE DEFINITION OF GEOMETRIC IMPERFECTIONS

1.1 Brief literature review

Early attempts to calculate the ultimate strength of plated and shell-like structures assumed an imperfection free geometry. Gradually, and based on experimental results, this assumption was dropped and it became evident the necessity of introducing initial imperfections. Among the studies that have considered an imperfect geometry are those of von Karman *et al.* [1], von Karman & Tsien [2], Koiter [3] and Budiansky & Hutchinson [4]. The last two are, in fact, considered to be the works proving that initial geometric imperfections are indeed the main reason for the poor correlation between theoretical and experimental results. Although these works were a turning point on the awareness of the importance of initial geometric imperfections, they assumed a constant pattern during the application of the load [5].
Nowadays it is possible to consider the variability of the imperfections shape during the analysis. This possibility came with the evolution of computer capacity allowing the use of advance numerical applications where the imperfect geometry is modelled and fully nonlinear analyses (GMNIA) are carried out.
According to the ECCS manual on design recommendations for shells [5], there are three conceptual approaches to define the initial geometric imperfections: realistic patterns, worst pattern and stimulating patterns.

1.2 European standards

As already mentioned, rules for defining equivalent geometric imperfections can be found in European standards.
For plated structures, imperfections are dealt in Annex C of EN1993-1-5 [6]. One possible approach (equivalent geometric imperfections) states that the imperfection shape may be defined following a relevant eigenmode shape or shapes defined in Figure C.1 with amplitudes given in Table C.2. In the case of unstiffened plates or sub-panels the amplitude proposed is

$$\Delta w_{0,eq,EN1993-1-5} = \min(a/200; b/200) \tag{1}$$

where $\Delta_{w0,eq,EN1993-1-5}$ is the maximum amplitude for the equivalent geometric imperfection,
a is the length of the plate (or sub-panel),
b is the width of the plate (or sub-panel).

For shells of revolution, recommendations to model equivalent geometric imperfections are given in section 8.7 of EN1993-1-6 [7]. The amplitude of the equivalent geometric imperfections is given by the following expression

$$\Delta w_{0,eq,EN1993-1-6} = \max(l_g U_n; 25 t U_n) \tag{2}$$

where $\Delta_{w0,eq,EN1993-1-6}$ is the maximum amplitude for the equivalent geometric imperfection,
l_g is the relevant gauge length according to clause 8.4.4(2),
U_n is the dimple imperfection amplitude parameter depending on the fabrication tolerance quality class,
t is the shell's thickness.

In conclusion, since cylindrically curved panels are a structural element that conceptually is limited by flat plates on one hand, and by full revolution shells on the other, there is a void in European standards in what concerns rules defining equivalent geometric imperfections.

2 NUMERICAL MODEL AND PARAMETRIC STUDY

2.1 General

The generic curved panel numerical model is the same as the one used in previous papers by the authors ([8] and [9]) and it is characterised by its curvature parameter, given by expression (3), it is simply supported along all its edges (loaded edges constrained and unloaded edges unconstrained in *y*-direction, see *Fig. 1*). The mechanical properties of the steel are given in *Table 1*.

$$Z = \frac{b^2}{R.t} \tag{3}$$

where Z is the non-dimensional curvature parameter,
b is the width of the curved panel,
R is the radius of curvature of the curved panel,
t is the curved panel's thickness.

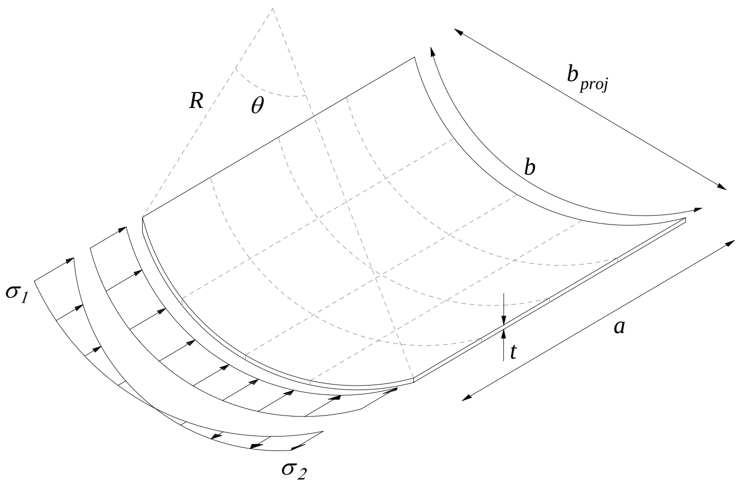

Fig. 1. Cylindrically curved panel [9]

Table 1. Mechanical properties of the steel (according to EN10025 [10])

Young's module, E	Poisson's coefficient, ν	Yield stress, f_y	Ultimate stress, f_u
210 GPa	0.3	t≤16 mm, 355 MPa	3<t≤80 mm, 470 MPa

2.2 Definition of the pattern of the equivalent geometric imperfections

As it will be seen at the definition of the parametric study, the patterns for the geometric patterns are based on the ten first buckling modes. This allows setting a comprehensive range for the initial shape of the geometric imperfections. As an example, these buckling modes are shown in *Fig. 2* for a cylindrically panel with a curvature parameter equal to 30 and aspect ratio (α=*a/b*) equal to 1.6.

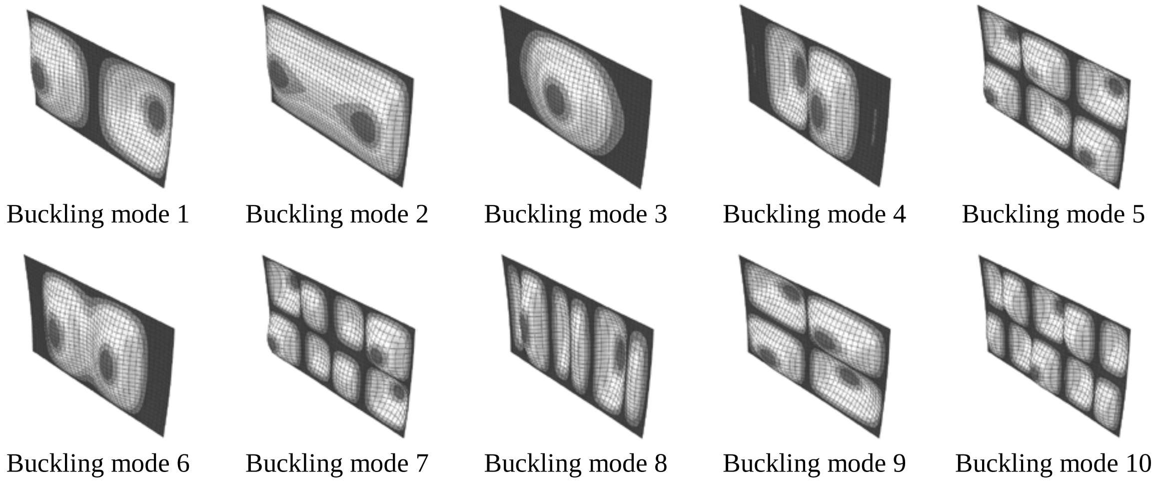

Fig. 2. Buckling modes for a cylindrically curved panel with Z=30 and α=1.6

2.3 Parametric study

A total of 1650 analyses were carried out. The range of all parameters intervening in this study is presented in *Table 2*. The parametric study comprises *b/t* ratios equal to 100, 150 and 200. These values are equivalent to non-dimensional slenderness parameters from 0.5 to approximately 2.5 (see [9]). This range is where imperfections play a more important role: for lower values of the non-dimensional slenderness parameter the ultimate strength is driven by plasticity and, for higher values of the non-dimensional slenderness parameters, the ultimate strength is driven by stability.

Table 2. Range of the parametric study

Width, b	Thickness, t	Curvature, Z	Aspect ratio	Imperfection shape and amplitude
1000 mm 1500 mm 2000 mm	10 mm	1, 10 to 100 step=100	1.0 to 5.0 step=1.00	10 buckling modes amp.=b/200

3 DISCUSSION OF RESULTS

3.1 Influence of imperfections' pattern on the postbuckling behaviour

The imperfection's pattern has a strong influence on the postbuckling path of cylindrically curved panes under pure compressive stresses. This statement is supported by *Fig. 3*. In fact, it is possible to see in *Fig. 3* that for the case where geometric imperfections are based on buckling mode no.3, the postbuckling path is characterised by an unstable part, while for buckling mode no.2 and no.6 the postbuckling path is stable until reaching the ultimate load.

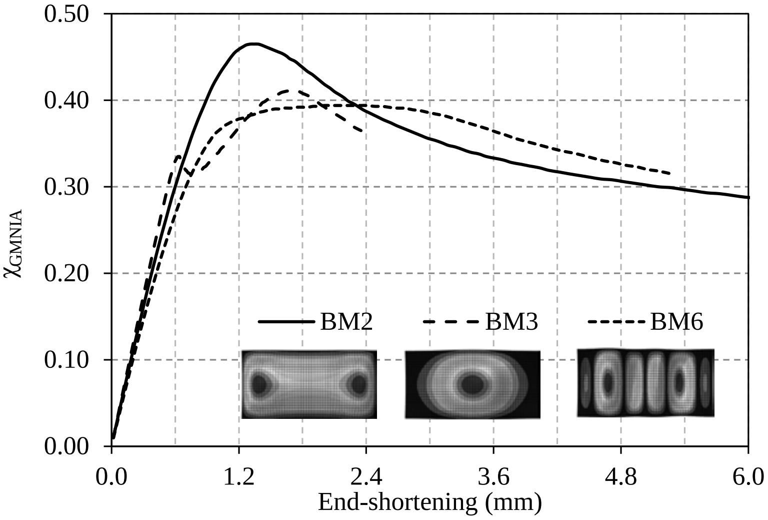

Fig. 3. Different postbuckling paths for a cylindrically curved panel characterised by Z=30 and α=2.0

Additionally, the pattern of initial geometric imperfections can dictate whether a cylindrically curved panel has an unstable postbuckling path highly sensitive to imperfections (resembling the postbuckling path of a shell of revolutions, *Fig. 4a)*) or a stable postbuckling path (resembling a postbuckling path of a plate, *Fig. 4b)*).

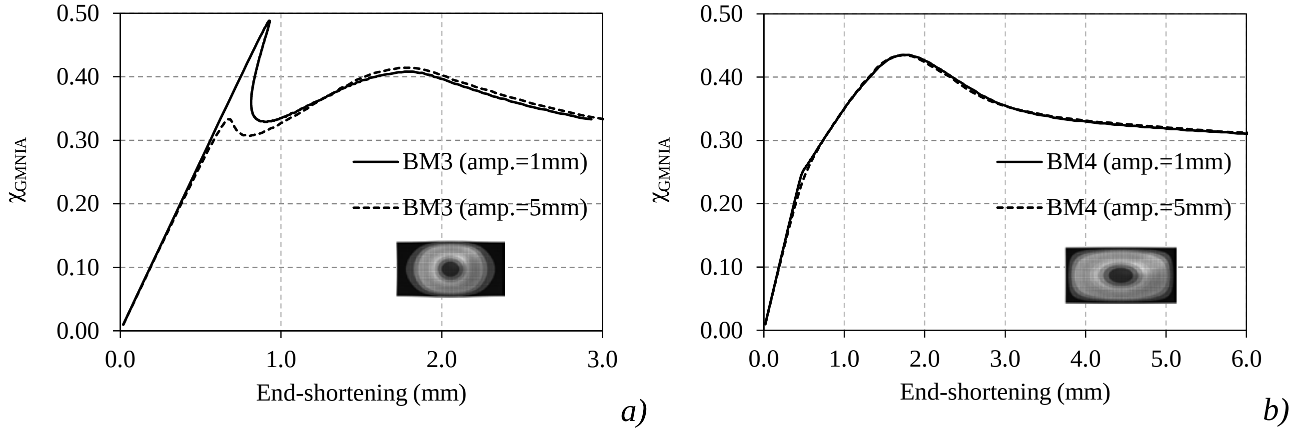

Fig. 4. Postbuckling path for a cylindrically curved panel characterised by: a) Z=30 and α=2.0; b) Z=10 and α=2.0

3.2 Influence of imperfections' pattern on the ultimate strength

The first buckling mode is usually used as imperfection shape. This is due the belief that, when used as imperfection shape, it will always yield the lowest ultimate load factor (as the first buckling mode is associated to the lowest bifurcation load and, therefore, associated to lowest energy necessary to change the state of equilibrium of a given system). This may be true for some types of structures (*e.g.* unstiffened plated structures), but it may also be an unsafe approach for others.

For example, *Fig. 5* shows the results for three different geometric configurations where the first buckling mode do not yield the minimum ultimate load factor once.

Additionally, *Table 3* shows the detailed results from those curved panels in *Fig. 5* and the differences between the ultimate load factors from panels with imperfections shape based on the first buckling mode and the remaining ones. For the curved panel characterised by Z=20 it is concluded that the first buckling mode returns the second highest value of the ultimate load factor.

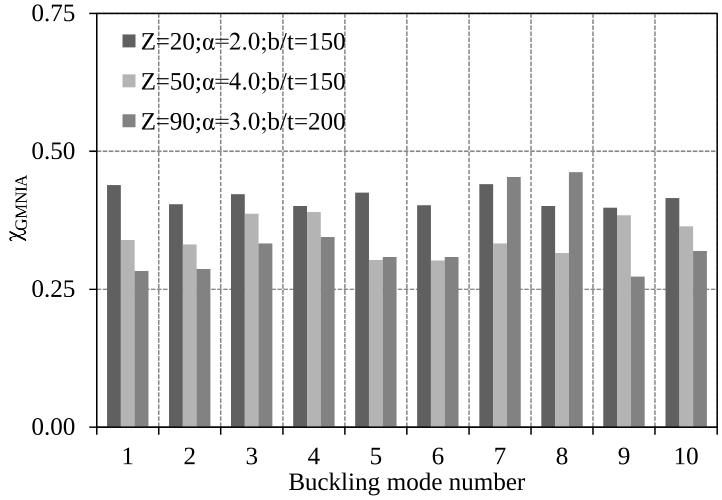

Fig. 5. Ultimate load factor for three different geometric configurations of cylindrically curved panels

Table 3. Comparison of results from models with imperfection shapes based on BM1 and the remaining buckling modes

	Z=20; *α*=2.0; *b/t*=100		*Z*=50; *α*=4.0; *b/t*=150		*Z*=90; *α*=3.0; *b/t*=200	
	ULF	$(ULF_{BM1}-ULF_{BMi})/ULF_{BM1}$	ULF	$(ULF_{BM1}-ULF_{BMi})/ULF_{BM1}$	ULF	$(ULF_{BM1}-ULF_{BMi})/ULF_{BM1}$
BM1	0.439	---	0.339	---	0.283	---
BM2	0.404	-8.0%	0.331	-2.4%	0.287	+1.4%
BM3	0.422	-3.9%	0.387	+14.2%	0.333	+17.7%
BM4	0.401	-8.7%	0.390	+15.0%	0.345	+21.9%
BM5	0.425	-3.2%	0.303	-10.6%	0.309	+9.2%
BM6	0.402	-8.4%	0.302	-10.9%	0.309	+9.2%
BM7	0.440	+0.2%	0.333	-1.8%	0.454	+60.4%
BM8	0.401	-8.7%	0.316	-6.8%	0.462	+63.3%
BM9	0.398	-9.3%	0.384	+13.3%	0.273	-3.5%
BM10	0.415	-5.5%	0.364	+7.4%	0.320	+13.1%

3.3 Effect of curvature on the ultimate strength of long cylindrically curved panels

Fig. 6 shows the evolution of the ultimate load factor with the curvature parameter.

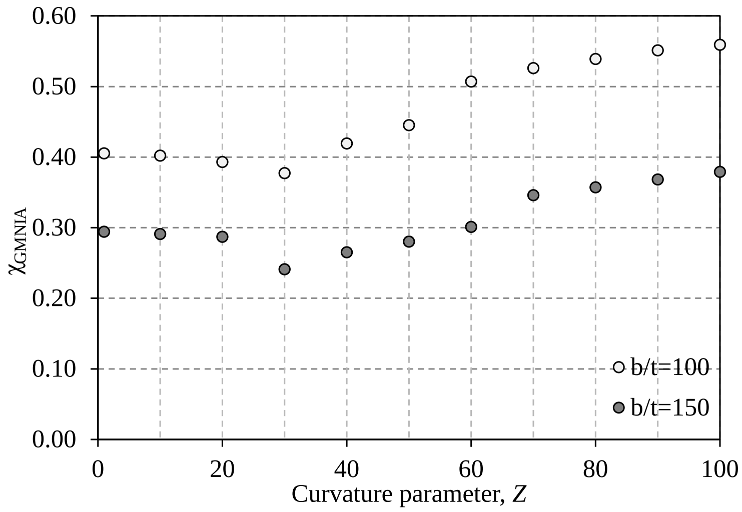

Fig. 6. Minimum values of the ultimate load factor for models with *b/t*=100 and 150

It is seen that, for long cylindrically curved panels, curvature has an unexpected effect: curved panels with curvature parameters around 30 have the lowest resistance. This means that, contrary to the conclusions for short cylindrically curved panels (see [9]), long curved panels are highly sensitive to initial geometric imperfections, especially for curvature parameters around 30.

4 CONCLUSIONS

In conclusion, this paper presents a preliminary study on the influence of the pattern of geometric imperfections on the ultimate behaviour of cylindrically curved panels. Namely, it was seen that, in what concerns geometric imperfections based on eigenmode shapes, these have a strong influence in both the postbuckling and ultimate strength of cylindrically curved panels.

The question of which is the worst shape for geometric imperfections (*i.e.* the one that results in the lowest value of the ultimate load factor for a given geometry) is still not answered. Nevertheless, the authors are currently involved in a study where it expected that this and other questions (like what is the effect of the geometric amplitude on the ultimate strength) will be answered.

5 ACKNOWLEDGMENT

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REFERENCES

[1] von Karman, T., Sechler, E. E., and Donnell, L. H. (1932), “Strength of thin Plates in Compression”, Trans. ASME, Vol. 54, No. APM-54-5, pp. 53-58.

[2] Karman T. H., Tsien H. S. (1941), “The buckling of thin cylindrical shells under axial compression”, Journal of the Aeronautical Sciences, Vol.8, No. 8, pp. 303-312.

[3] Koiter, W. T. (1945), “Over de Stabiliteite van het Elastische Evenwicht”, Ph.D. Thesis, Delft University (English trans.in 1967:” On the stability of elastic equilibrium”, NASA Technical Translation No. TT F 10833, Washington D. C.).

[4] Budiansky, B., Hutchinson, J. W. (1964), “Dynamic Buckling of Imperfection Sensitive Structures”, Proceedings 11th IUTAM Congress, Springer Verlag, Berlin, pp. 636-651.

[5] ECCS (2008), “Buckling of shells, European Design recommendations”, Publication No. 128, 5th ed., European Convention for Constructional Steelwork, Rotter J. M., Schmidt, H. (Eds.), Brussels.

[6] CEN (2006), EN1993-1-5:2006 - Eurocode 3: Design of steel structures, Part 1-5: Plated Structural Elements, European Committee for Standardization, Brussels.

[7] CEN (2007), EN1993-1-6:2007 - Eurocode 3: Design of steel structures, Part 1-6: Strength and Stability Shells Structures, European Committee for Standardization, Brussels.

[8] Martins, J. P., Simões da Silva, L. and Reis, A. (2013), “Eigenvalue analysis of cylindrically curved panels under compressive stresses – Extension of rules from EN1993-1-5”, Thin-Walled Structures, vol. 68, pp. 183-194.

[9] Martins, J. P., Simões da Silva, L. and Reis, A. (2013), “Ultimate load of cylindrically curved panels under in plane compression and bending – Extension of rules from EN1993-1-5”, Thin-Walled Structures, vol. 77, pp. 36-47.